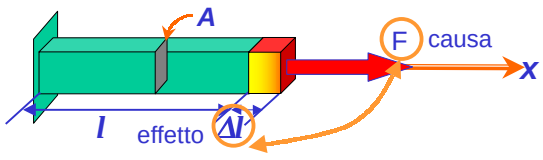
Relazione tensione-deformazione

Relazione tensione-deformazione

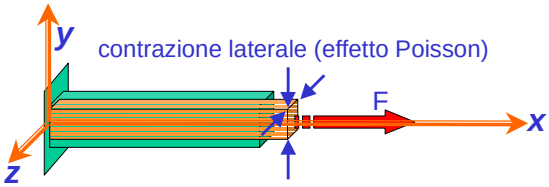
Relazioni costitutive in campo elastico



$F = K\,\Delta l$ dove $K = \frac{A\,E}{l}$

quindi $\Delta l = \frac{F\,l}{A\,E} = \frac{\sigma_x\,l}{E}$

$\frac{\Delta l}{l} = \frac{\sigma_x}{E}$ $\quad \varepsilon_x = \frac{\sigma_x}{E}$



$\varepsilon_y = \frac{\sigma_y}{E}$ $\quad \varepsilon_z = \frac{\sigma_z}{E}$

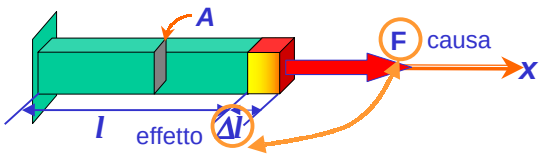
$\varepsilon_y = -\nu\frac{\sigma_x}{E}$ $\quad \varepsilon_z = -\nu\frac{\sigma_x}{E}$

nel caso più generale di stato di tensione triassiale la deformazione ε_x è data da:

$$\varepsilon_x = 1/E\,[\sigma_x - \nu\,(\sigma_y + \sigma_z)]$$

Relazione tensione-deformazione

Relazioni costitutive in campo elastico



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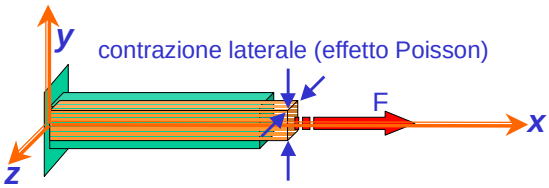


$\varepsilon_y = \frac{\sigma_y}{E}$ $\quad \varepsilon_z = \frac{\sigma_z}{E}$

In modo analogo si possono ottenere le ε_y ed ε_z :

$\varepsilon_y = -\nu \frac{\sigma_x}{E}$ $\quad \varepsilon_z = -\nu \frac{\sigma_x}{E}$

$\varepsilon_y = 1/E\,[\sigma_y - \nu(\sigma_x + \sigma_z)]$

$\varepsilon_z = 1/E\,[\sigma_z - \nu(\sigma_x + \sigma_y)]$

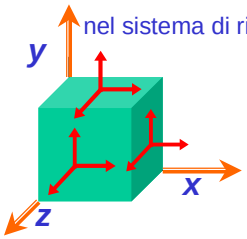
Relazione tensione-deformazione

Relazioni costitutive in campo elastico

stato di tensione triassiale

relazione costitutiva in campo elastico tra deformazione ε e la tensione σ :

nel sistema di riferimento xyz

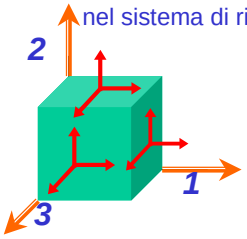


$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$

$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$

$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$

nel sistema di riferimento principale



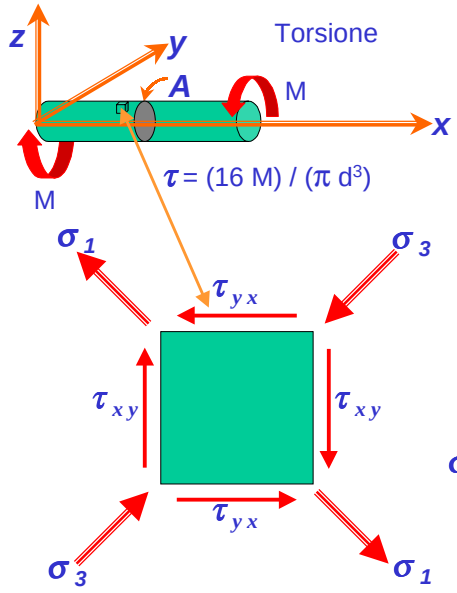
$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$

$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$

$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \nu(\sigma_1 + \sigma_y)]$

Relazione tensione-deformazione

Relazioni costitutive in campo elastico



Nel caso di torsione pura, con la disposizione degli assi scelta, si ha:

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$$

$$\tau_{xy} \neq 0$$

L'equazione cubica dello stato di tensione assume quindi la seguente forma:

(si veda anche il cerchio di Mohr nel caso di torsione pura)

$$\sigma^3 - \sigma \tau_{xy}^2 = 0$$

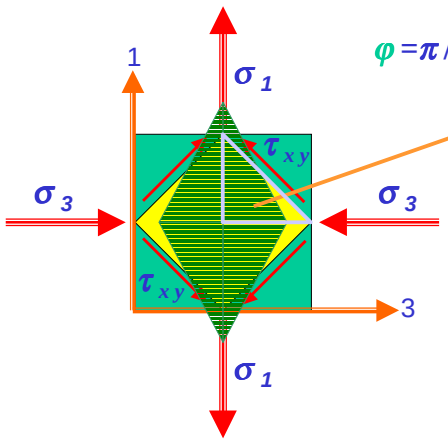
da cui si ottiene:

$$\sigma_3 = -\sigma_1$$

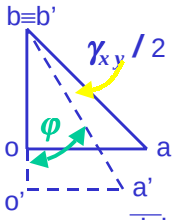
$$\sigma_1 = \tau_{xy}$$
$$\sigma_2 = 0$$
$$\sigma_3 = -\tau_{xy}$$

Relazione tensione-deformazione

Relazioni costitutive in campo elastico



$$\varphi = \pi/4 - \gamma_{xy}/2$$



$$tg(\pi/4 - \gamma_{xy}/2) = \frac{\overline{o'a'}}{\overline{o'b'}}$$

$$\frac{\overline{o'a'}}{\overline{o'b'}} = \frac{\overline{oa}\,(1+\varepsilon_3)}{\overline{oa}\,(1+\varepsilon_1)} \qquad \frac{\overline{oa}}{\overline{ob}} = 1$$

$$tg(\pi/4 - \gamma_{xy}/2) = \frac{(1+\varepsilon_3)}{(1+\varepsilon_1)}$$

$$tg(\alpha - \beta) = \frac{tg\alpha - tg\beta}{(1+tg\alpha\ tg\beta)} \Rightarrow tg(\pi/4 - \gamma_{xy}/2) = \frac{tg\ \pi/4 - tg\ \gamma_{xy}/2}{(1+tg\ \pi/4\ \ tg\ \gamma_{xy}/2)}$$

Relazione tensione-deformazione

Relazioni costitutive in campo elastico

$$tg(\pi/4 - \gamma_{xy}/2) = \frac{tg\,\pi/4 - tg\,\gamma_{xy}/2}{(1+ tg\,\pi/4\ tg\,\gamma_{xy}/2)} = \frac{1 - \gamma_{xy}/2}{1+ \gamma_{xy}/2} = \frac{(1+\varepsilon_3)}{(1+\varepsilon_1)}$$

($tg\,\pi/4 = 1$; $tg\,\gamma_{xy}/2 \cong \gamma_{xy}/2$)

$$\sigma_3 = -\sigma_1 \Rightarrow \varepsilon_3 = -\varepsilon_1 \Rightarrow \frac{1 - \gamma_{xy}/2}{1+ \gamma_{xy}/2} = \frac{(1-\varepsilon_1)}{(1+\varepsilon_1)}$$

essendo:

$$\varepsilon_1 = 1/E\,[\sigma_1 - \nu(\sigma_2+\sigma_3)] \quad \Leftarrow \quad \gamma_{xy} = 2\varepsilon_1$$

si ottiene:

$$\gamma_{xy} = 2/E\,[\sigma_1 - \nu(\sigma_2+\sigma_3)]$$

nel caso della torsione pura $\sigma_1 = \tau_{xy}$ e $\sigma_3 = -\tau_{xy}$ e quindi:

$$\gamma_{xy} = 2(1+\nu)/E\ \sigma_1 = 2(1+\nu)/E\ \tau_{xy} = 1/G\ \tau_{xy} \rightarrow G = E/[2(1+\nu)]$$

Relazione tensione-deformazione

Relazioni costitutive in campo elastico

Sollecitazione di taglio:

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \gamma_1 = \frac{\tau_1}{G} \qquad \gamma_1 = \frac{\sigma_2 - \sigma_3}{2G} \qquad \gamma_1 = \varepsilon_2 - \varepsilon_3$$

$$\gamma_{xz} = \frac{\tau_{xz}}{G} \qquad \gamma_2 = \frac{\tau_2}{G} \qquad \gamma_2 = \frac{\sigma_1 - \sigma_3}{2G} \qquad \gamma_2 = \varepsilon_1 - \varepsilon_3$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \gamma_3 = \frac{\tau_3}{G} \qquad \gamma_3 = \frac{\sigma_1 - \sigma_2}{2G} \qquad \gamma_3 = \varepsilon_1 - \varepsilon_2$$

Relazione tensione-deformazione

Relazioni costitutive in campo elastico

valida in campo elastico (per i materiali isotropi)

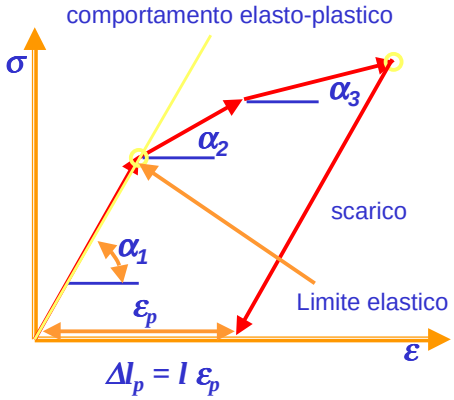
$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \left[\begin{array}{ccc|ccc} 1/E & -\nu/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & 1/E & -\nu/E & 0 & 0 & 0 \\ -\nu/E & -\nu/E & 1/E & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1/G & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G \end{array}\right] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}$$

Le sottomatrici nulle indicano che non c'è accoppiamento tra componenti normali della tensione e distorsione

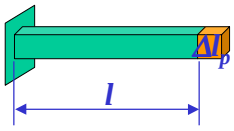
Relazione tensione-deformazione

Come si comporta il materiale oltre il limite elastico?

comportamento elasto-plastico



$\Delta l_p = l\ \varepsilon_p$

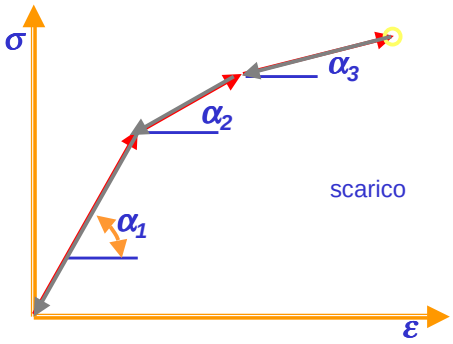


Rimuovendo la forza applicata rimane un allungamento residuo permanente

Relazione tensione-deformazione

Come si comporta il materiale oltre il limite elastico?

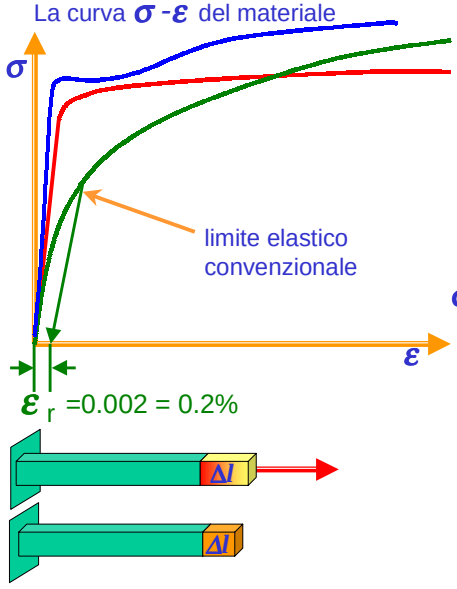
comportamento elastico non lineare

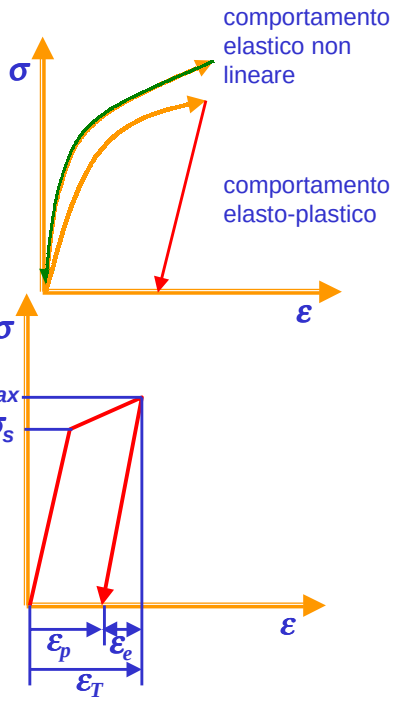


Al termine del ciclo di carico non rimane alcuna deformazione permanente

Relazione tensione-deformazione

La curva σ-ε del materiale

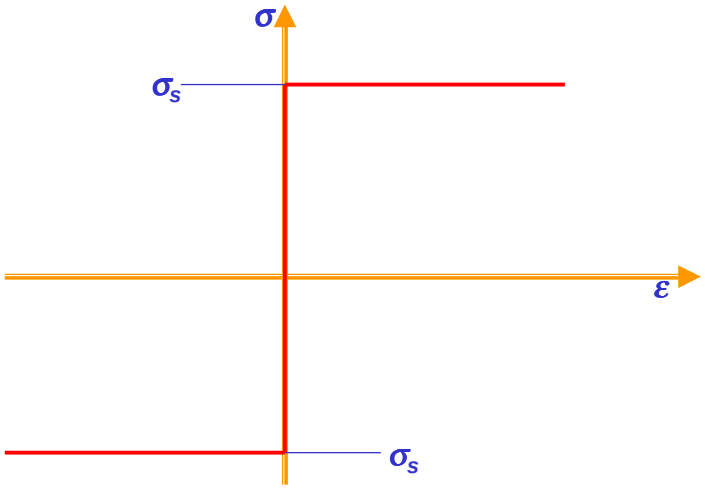




Relazione tensione-deformazione

La curva σ - ε

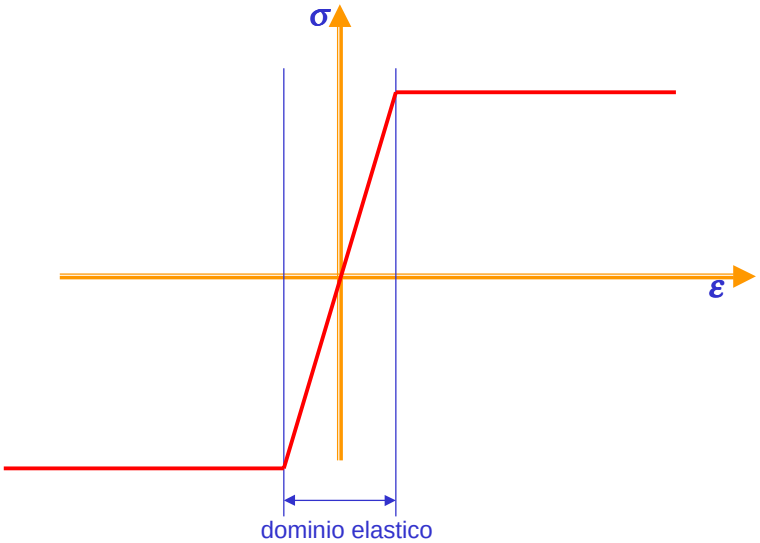
Modello bilineare: rigido-plastico



Relazione tensione-deformazione

La curva σ - ε

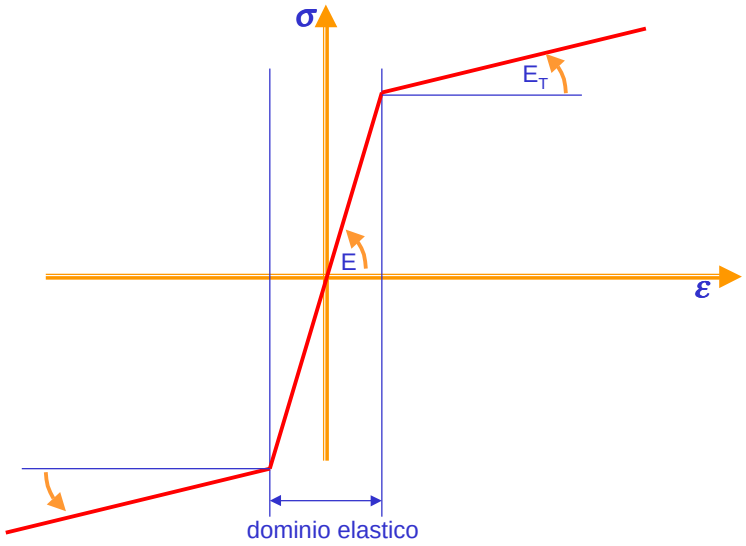
Modello bilineare: elasto-plastico perfetto



Relazione tensione-deformazione

La curva σ - ε

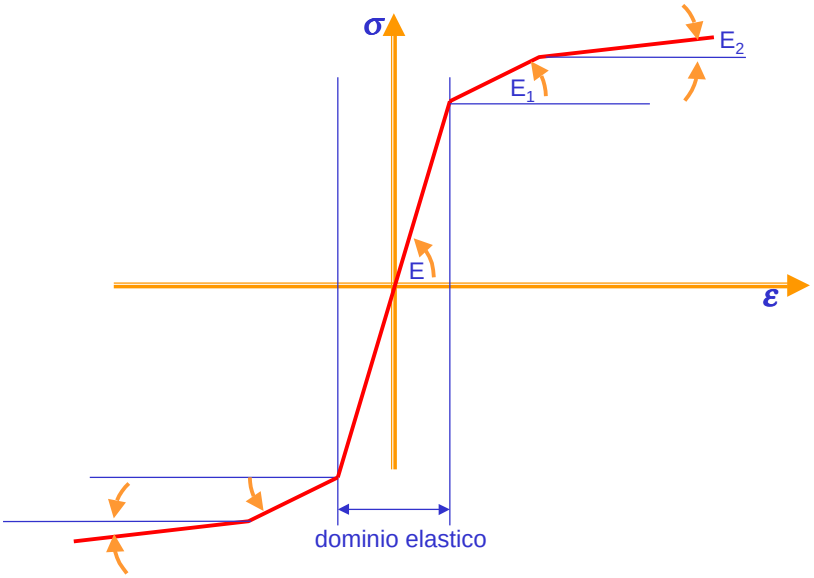
Modello bilineare: elasto-plastico con incrudimento



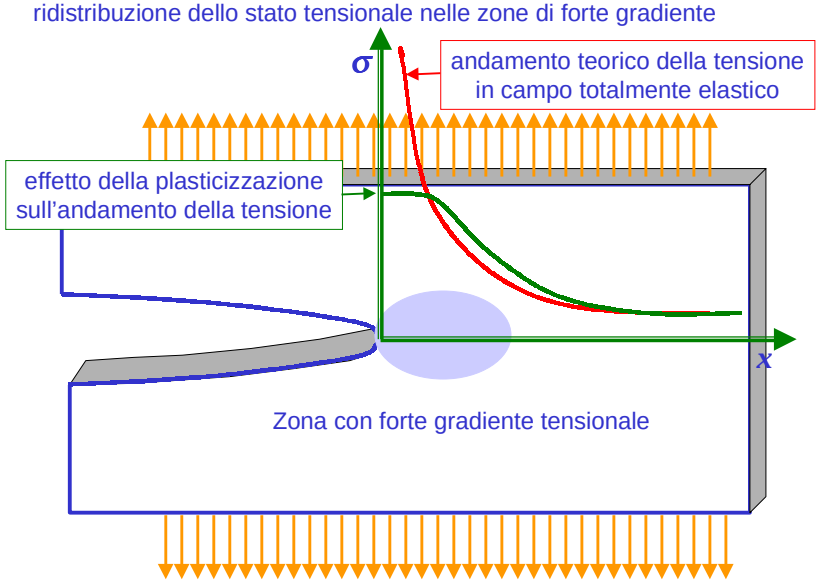
Relazione tensione-deformazione

La curva σ - ε

Modello multilineare: elasto-plastico con incrudimento



Relazione tensione-deformazione

La curva σ - ε

Effetto benefico del comportamento plastico:
ridistribuzione dello stato tensionale nelle zone di forte gradiente